Estimating pi

As pi is an irrational number, many people have spent time attempting to find the best way of estimating pi. Here are a few examples of men who came up with different ways of doing this.

James Gregory (1638-1675)

James Gregory came up with an infinite series which could calculate pi. His series looked like this:

$$\pi = 4 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

This series is not very accurate. It would take 5 million terms of the series to obtain pi correct to 6 or 7 decimals. There was a need for a more accurate way of approximating pi.

John Machin

John Machin was able to come up with a much more accurate series:

$$\frac{\pi}{4} = 4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$

Machin's approximation was so good, it is still being used by computer programmers as an approximation to pi. William Shanks actually used this approximation to calculate pi to 707 decimal places.

Leonhard Euler

Euler was born in Basel in 1707 and contributed much to the world of Mathematics in his time. He worked on many things such as analytic gemoetry, trigonometry, differential equations and mechanics. He also introduced new notation such as f(x), e (for the exponential function) and i (to represent the square root of -1). For more on Euler click here. He derived this equation for pi:

$$\pi = \sqrt{6\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\right)},$$

10001 terms of this series will give an aprroximation to pi with accuracy of 9×10^{-5}. This is a very good approximation of pi but was nothing compared to the approximation produced by the next man.

Srinivasi Ramanujan

Ramanujan was born in India in 1887. Despite very little formal mathematical training, he made many contributions to number theory and infinite series among other things. For more about Ramanujan click here. His estimate of pi looked like this:

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2} + \frac{27493}{99^6}\frac{1}{2}\frac{1 \cdot 3}{4^2} + \frac{53883}{99^{10}}\frac{1 \cdot 3}{2 \cdot 4}\frac{1 \cdot 3 \cdot 5 \cdot 7}{4^2 \cdot 8^2} + \ldots$$

This series is amazing in that it only needs 2 terms and the approximation is already pi accurate to 9×10^{-16}. Not only that, but this example of Ramanujan's work is typical having appeared from nowhere and with no proof.

George Conte de Buffon (1707-1788)

The approximation given so far have been infinite series' but Buffon's approximation was different in that it dealt with probability. Here is the situation: a floor is covered in floor-boards of width a and you have a pin of length a which you throw randomly onto the floor. The probability that the pin lands and cuts one of the lines between the floor boards is p. Buffon proved that this probability was equal to $2/\pi$! So to estimate pi, Buffon carried out the experiment, calculated the ratio of the number of pins that cut a line to the number of trials and used this estimated probability to calculate pi. To view a simulation of this click here.
